Abstract

Let $G$ be a finite group. Artin’s theorem says that we can recover the complex representation ring of $G$ from the representations of the cyclic subgroups of $G$ up to torsion, or additive nilpotence. Quillen’s $F$-isomorphism theorem says that we can recover the mod-$p$ cohomology of $G$ from the mod-$p$ cohomology of the elementary abelian $p$-subgroups of $G$ up to multiplicative nilpotence.

These results naturally fit into Dress’s theory of induction/restriction for Mackey and Green functors. These particular Mackey functors arise as the homotopy groups of $G$-spectra and it is relatively straightforward to lift Dress’s framework to $G$-spectra. In this derived framework we associate to each family $F$ of subgroups of $G$ a subcategory $F$-nil, of $G$-spectra with the following properties: If $E \in F$-nil then: 1) A generalized version of Artin’s induction/restriction theorem holds for $E$-equivariant (co)homology. 2) If $E$ is a homotopy commutative ring spectrum, then a generalization of Quillen’s $F$-isomorphism theorem holds for $E$-equivariant cohomology of $G$-spaces. 3) If $E$ is an $E_{\infty}$ ring spectrum then we can recover the category of $G$-equivariant $E$-modules from the categories of $H$-equivariant $E$-modules as $H$ varies over $F$.

Moreover the category $F$-nil is closed under finite homotopy (co)limits, retracts, and (co)tensors with arbitrary $G$-spectra. Our theory applies to genuine equivariant complex and real K-theory (extending Artin’s theorem and a result of Fausk), and the Borel equivariant cohomology theories associated to mod-$p$ cohomology (extending...
the result of Quillen), integral cohomology (extending a result of Carlson), complex oriented theories (extending a result of Hopkins-Kuhn-Ravenel), ko, the many variants of topological modular forms, $L_n$-local spectra, and classical cobordism theories.

This is joint work with Akhil Mathew and Niko Naumann.