Homotopical torsor theory and complex orientations of algebraic $K$-theory

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Abstract

If $L/K$ is a Galois extension of fields and $M, N$ are two $K$-vector spaces (or $K$-algebras, or $K$-schemes, ...) then we say that $M$ is an $L/K$-twisted form of $N$ if $M$ and $N$ become isomorphic after tensoring them each up to $L$. When $M$ is an algebraic gadget of some kind over $K$ which becomes a free such gadget after tensoring up to $L$, we sometimes say that $M$ is an $L/K$-torsor.

In this talk we develop some torsor theory applicable to homotopy theory. We work in the general setting of symmetric monoidal stable model categories, and we develop Bousfield localization, nilpotent completion, the Adams and Adams-Bruner spectral sequences, and Bousfield’s comparison theorems relating the localization-to-completion comparison map to horizontal vanishing lines in the Adams spectral sequence, all in a very wide level of generality. Then we use this theory to give a very complete analysis of the homotopy types of torsors and their localizations and completions.

As an application we give a global (no localizing at a prime!) computation of the $KU$-local stable homotopy type of infinite-dimensional complex projective space, and as a consequence, this implies some positive and some negative results on when the algebraic $K$-theory spectrum of a scheme is complex-orientable.