MEASURABLE SELECTORS, PROXIMINALITY AND INTEGRATION OF MULTI-FUNCTIONS

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ABSTRACT. Kuratowski and Ryll-Nardzewski's selection theorem is a magnificent tool to obtain measurable selectors of certain multi-functions; one of the drawbacks of this result is that separability is required for the range space. In this lecture we will show how to use descriptive set-theoretic techniques to overcome the above separability assumption and as application obtain that $L^1(\mu, Y)$ is proximinal in $L^1(\mu, X)$ when $Y \subset X$ is a proximinal subspace and the Banach space X is *nice*, for instance WCG. We will present our advances when studying the existence of measurable selectors for multi-functions whose values are weakly compact subsets of a Banach space *without separability* assumptions about the range space: on one hand, we characterize multi-functions having strongly measurable selectors; on the other hand, we prove that every scalarly measurable multi-function admits scalarly measurable selectors. By doing so we solve an open problem in the area and extend the theory of Pettis integration for multi-functions that previously was only known in the separable case to the case of general Banach spaces. The parallelism between the techniques presented and some questions arising from topology will be presented.

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