A Homotopy-Theoretical Approach to the Sullivan Conjecture

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Abstract

In the early 1970s, Dennis Sullivan conjectured that for any finite-dimensional space $K$, the space of pointed maps $\text{map}_*(B\mathbb{Z}/p, K)$ must be weakly contractible: all of its homotopy groups must be zero.

In the early 1980s, Haynes Miller proved this conjecture by developing, essentially from scratch, huge bunches of very difficult pure algebra. The algebraic techniques led to the development of the $T$ functor, which is now a central tool in the vast and expanding study of the Steenrod algebra and its action on the cohomology of spaces.

In this talk, I’ll show how most of this could have been avoided and the original conjecture proved in the early 1970s using theorems which were, by then, classical. The trick is to introduce a new point of view: a kind of Galois theory for collections of spaces.